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CONTRIBUTIONS FROM THE WILDER PHYSICAL LABORATORY
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THE PRESSURE DUE TO RADIATION.

By E. F. Nichols and G. F. Hull.

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Contents.

As early as 1619 Kepler * announced his belief that the solar repulsion of the finely divided matter of comets' tails was due to the outward pressure of light. On the corpuscular theory of light Newton † considered Kepler's idea as plausible enough, but he was of the opinion that the phenomenon was analogous to the rising of smoke in our own atmosphere. In the first half of the eighteenth century DeMairan and DuFay ‡ contrived elaborate experiments to test this pressure of light

* DeMairan, Traité physique et historique de l'Aurore Boréale (Seconde Edition), pp. 357–358. Paris, 1754.

† Isaaci Newtoni Opera quae Existant Omnia. Samuel Horsley, LL.D., R. S. S. Tom. III., pag. 156, Londinium, 1782.

‡ DeMarian, l. c., p. 371. This treatise contains also the accounts of still earlier experiments by Hartsoeker, p. 368, and Homberg, p. 369. The later experiments are of more historic than intrinsic interest.

theory in the laboratory, but, because of the disturbing action of the gases surrounding the illuminated bodies employed in the measurements, they obtained wholly confusing and contradictory results. Later in the same century the Rev. A. Bennet* performed further experiments, but could find no repulsive force not traceable to convection currents in the gas surrounding the body upon which the light was projected, due in his opinion to the heating effect of the rays. Finding no pressure due to radiation, he made the following unique suggestion in support of the wave theory of light: "Perhaps sensible heat and light may not be caused by the influx or rectilinear projection of fine particles, but by the vibrations made in the universally diffused caloric or matter of heat or fluid of light. I think modern discoveries, especially those of electricity, favor the latter hypothesis." In the meantime Euler,† accepting Kepler's theory attributing the phenomenon of comets' tails to light pressure, had hastened to the support of the wave theory by showing theoretically that a longitudinal wave motion might produce a pressure in the direction of its propagation upon a body which checked its progress. In 1825 Fresnel ‡ made a series of experiments, but arrived at no more definite conclusion than that the repulsive and attractive forces observed were not of magnetic nor electric origin.

Crookes § believed in 1873 that he had found the true radiation pressure in his newly invented radiometer and cautiously suggested that his experiments might have some bearing on the prevailing theory of the nature of light. Crookes' later experiments and Zöllner's ‖ measurements of radiometric repulsions showed that the radiometric forces were in some cases 100,000 times greater than the light pressure forces with which they had been temporarily confused. Zöllner's experiments are among the most ingenious ever tried in this field of work, and he missed the discovery of the true radiation pressure by only the narrowest margin. An excellent bibliography of the whole radiometric literature is given by Graetz,¶ and an account of some of the older experiments not mentioned above is given by Crookes.**

* A. Bennet, Phil. Trans., p. 81 (1792).

† L. Euler, Histoire de l'Academie Royale de Berlin (2), p. 121 (1746).

‡ A. Fresnel, Ann. Chem. et Phys., XXIX. 57, 107 (1825).

§ W. Crookes, Phil. Trans., p. 501 (1873).

‖ F. Zöllner, Pogg. Ann., CLX. 156, 296, 459 (1877).

¶ L. Graetz, Winckelmann's Handbuch der Physik, 2b, p. 262. Breslau, 1896.

** W. Crookes, l. c., p. 501.

In 1873 Maxwell,* on the basis of the electromagnetic theory, showed, that if light were an electromagnetic phenomenon, pressure should result from the absorption or reflection of a beam of light. After a discussion of the equations involved, he says: "Hence in a medium in which waves are propagated there is a pressure in the direction normal to the waves and numerically equal to the energy in unit volume." Maxwell computed the pressure exerted by the sun on the illuminated surface of the earth and added: "It is probable that a much greater energy of radiation might be obtained by means of the concentrated rays from an electric lamp. Such rays falling on a thin metallic disc, delicately suspended in a vacuum, might perhaps produce an observable mechanical effect."

Apparently independent of Maxwell, Bartoli † announced in 1876 that the Second Law of Thermodynamics required the existence of a pressure due to radiation numerically equal in amount to that derived by Maxwell. Bartoli's reasoning holds for all forms of energy streams in space and is of more general application than Maxwell's equations. Bartoli contrived elaborate experiments to verify this theory, but was balked in the search, as all before him had been, by the complicated character of the gas action which he found no way of eliminating from his experiments.

After Bartoli's work the subject was dealt with theoretically by Boltzmann,‡ Galitzine,§ Guillaume, ‖ Heaviside,¶ and more recently by Goldhammer.** Fitzgerald, †† Lebedew,‡‡ and Hull §§ have discussed the bearing of radiation pressure upon the Newtonian law of gravitation with special reference to the repulsion of comets' tails by the sun. Arrhenius ‖‖ has recently discussed the cosmical consequences of radiation pressure not only concerning comets' tails, but, by combining radiation pressure with the known properties of negative ions, has endeavored also to account

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* J. C. Maxwell, A Treatise on Electricity and Magnetism (1st Edition), II. 391. Oxford, 1873.

† A. Bartoli, Sopra i movementi prodotti della luce e dal calorie, Florence, Le Monnier (1876), also Nuovo Cimento, XV. 193 (1884).

‡ L. Boltzmann, Wied. Ann., XXII. 31, 291 (1884).

§ B. Galitzine, Wied. Ann., XLVII. 479 (1892).

‖ Ch. Ed. Guillaume, Arch. de Gen. (3), XXXI. 121 (1894).

¶ O. Heaviside, Electromagnetic Theory, I. 334. London, 1893.

** D. A. Goldhammer, Ann. Phys., IV. 834 (1901).

†† G. F. Fitzgerald, Proc. Roy. Soc. Dub. (1884).

‡‡ P. Lebedew, Wied. Ann., XLV. 292 (1892). Astrophys. Jour., XIV. 155 (1902).

§§ G. F. Hull, Trans. Astron. Soc. Toronto, p. 123 (1901).

‖‖ S. Arrhenius, Konigl, Vetanskaps. Akademiens Fördhandlingar, p. 545 (1900).

for the aurora borealis. Swartzschild* computed from radiation pressure on small spherical conductors the size of bodies of unit density for which the ratio of radiation pressure to gravitational attraction would be a maximum.

Before the Congrès International de Physique in 1900, Professor Lebedew† of the University of Moscow described an arrangement of apparatus which he was using at that time for the measurement of light pressure. He summarizes the results already obtained as follows: "Les résultats des mesures que j'ai faites jusqu'ici peuvent se résumer ainsi: L'expérience montre qu'un faisceau lumineux incident exerce sur les surfaces planes absorbantes et réfléchissantes des pressions qui, aux erreurs près d'observation, sont égales aux valeurs calculées par Maxwell et Bartoli." No estimate of the "errors of observation" was given in the paper nor other numerical data. Unfortunately the proceedings of the Paris Congress did not reach the writers nor any intimation of the methods or results of Professor Lebedew's work until after the publication of their own preliminary experiments.

The writers‡ presented the results they had obtained by measurements of radiation pressure at eight different gas pressures, in a preliminary communication to the American Physical Society, meeting with Section B of the American Association at Denver, August 29, 1901. A condensed abstract of this paper follows.

In the experiments of earlier investigators every approach to the experimental solution of the problem of radiation pressure had been balked by the disturbing action of gases which it is impossible to remove entirely from the space surrounding the body upon which the radiation falls. The forces of attraction or repulsion, due to the action of gas molecules, are functions, first, of the temperature difference between the body and its surroundings, caused by the absorption by the body of a portion of the rays which fall upon it; and second, of the pressure of the gas surrounding the illuminated body. In the particular form of apparatus used in the present study the latter function appears very complicated, and certain peculiarities of the gas action remain inexplicable upon the basis of any simple group of assumptions which the writers have so far been able to make.

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* K. Swartschild, Kgl. Bayer. Akademie d' Wissenschaften, XXXI. 293 (1901).

† P. Lebedew, Rapports présentés au Congrès International de Physique (2), p. 133. Paris, 1900.

‡ E. F. Nichols and G. F. Hull, Science, XIV. 588 (Oct. 18, 1901), and Phys. Rev., XIII. 293 (Nov., 1901).

Since we can neither do away entirely with the gas nor calculate its effect under varying conditions, the only hopeful approach which remains is to devise apparatus and methods of observation which will reduce the errors due to gas action to a minimum. The following considerations led to a method by which the elimination of the gas action was practically accomplished in the present experiments.

1. The surfaces which receive the radiation, the pressure of which is to be measured, should be as perfect reflectors as possible. This will reduce the gas action by making the rise of temperature due to absorption small, while the radiation pressure will be increased; the theory requiring that a beam, totally reflected, exert twice the pressure of an equal beam, completely absorbed.

2. By studying the action of a beam of constant intensity upon the same surface surrounded by air at different pressures, certain pressures may be found where the gas action is less than at others.

3. The apparatus — some sort of torsion balance — should carry two surfaces symmetrically placed with reference to the rotation axis, and the surfaces on the two arms should be as nearly equal as possible in every respect. The surfaces or vanes should be so constructed that if the forces due to gas action (whether suction or pressure on the warmer surface) and radiation pressure have the same sign in one case, a reversal of the suspension should reverse the gas action and bring the two forces into opposition. In this way a mean of the forces on the two faces of the suspension should be, in part at least, free from gas action.

4. Radiation pressure, from its nature, must reach its maximum value instantly, while observation has shown that gas action begins at zero and increases with length of exposure, rising rapidly at first, then more slowly to its maximum effect, which, in many of the cases observed, was not reached until the exposure had lasted from two and a half to three minutes. For large gas pressures, an even longer exposure was necessary to reach stationary conditions. The gas action may be thus still further reduced by a ballistic or semi-ballistic method of measurement.

The results of ballistic observations of radiation pressure at different gas pressures are given below in Table I, in which p indicates the pressure of the surrounding gas in millimeters of mercury, and d the static equivalent of the ballistic throws of the torsion balance. The results were obtained with substantially the same apparatus and method described on page 568 et seq.: —

TABLE I.

p	d	p	d
96.3	19.7	33.4	21.1
67.7	21.0	1.2	20.9
37.9	21.6	0.13	26.8
36.5	22.1	0.06	23.2
Mean 22.5			

Taking the product of the average deflection in centimeters by the constant of the torsion balance, the radiation pressure was: —

$$2.25 \times 4.65 \times 10^{-5} = 1.05 \times 10^{-4} \text{ dynes.}$$

The Bolometer.

To compare the theoretical value of radiation pressure with the above value, it was necessary to measure the energy of the radiation causing the pressure. This was attempted with the aid of a bolometer constructed as follows: —

On a sheet of platinum 0.001 mm. thick, rolled in silver (by the firm Sy & Wagner, Berlin), a circle P (Fig. 1), 11.25 mm. in diameter, was drawn. The sheet was cut from the edges inward to the circumference of the circle, in such a way as to leave five principal strips $A, B, C, D, E,$ connected to the circle in the manner shown. Other narrower strips, as $e, m, n, o,$ etc., were left to give the disc additional support. The disc, by means of the connecting arms, was mounted with asphalt varnish centrally over a hole, 14 mm. in diameter, bored through a slab S of thin slate. Portions of the silver not to be removed by the acid were carefully covered by asphalt varnish. Thus on the strips A and B, the silver was protected to the very edge of the circle, while on all the other arms, the silver was left exposed back to the edge of the boring in the slate. The whole system was then plunged into warm nitric acid, and the silver eaten away from all unvarnished surfaces, leaving only the thin platinum sheet which was blackened by electric deposition of platinum by Kurlbaum's* method. At $A, B, C, D, E,$ holes were bored extending

* Kurlbaum, Wied. Ann., LXVII. 848 (1899).

through the slate. Copper washers were soldered to the silver strips and binding posts were attached.

The torsion balance was removed from under the bell-jar, the bolometer was put in the place of one of the vanes and was covered by the bell-jar. Connections to the bolometer were made as schematically shown in Fig. 1. The disc P was the exact size of the light image thrown on the vanes in the pressure measurements. The intention was to heat the disc by allowing the image to fall on it, and then, with the light turned off, to heat it to the *same temperature* by sending a current through it from A to B. If r be the resistance from A to B in ohms, when exposed to the lamp, and i be the current in ampères which gives the same temperature in P as that given by the absorbed radiation, then $i^2r \times 10^7$ will be the activity of the beam in erg-seconds. The temperature of the disc, whether exposed to the radiation or heated by the current, was shown by the resistance, C to D-E, which was made one arm of a Wheatstone Bridge. The relation of the heating current to the bridge was adjusted as follows: With the key K open, so that no current flowed through the bridge, the heating current from six storage cells B_2 was turned on, and the sliding contact at F so set that the bridge galvanometer zero was not changed by reversing the heating current. The point F, equipotential to c, was found very near the middle of the wire $a\,b$, which showed the current distribution of P to be symmetrical with respect to a diameter at right angles to $A\,B$. The key K was then closed making the bridge current, and the bridge was balanced. The bolometer was next exposed to the radiation, and simultaneous observations of the intensity of the beam were made on galvanometer G_1 (Fig. 1), and the lamp galvanometer G_2. The deflection of galvanometer G_1 was reduced to standard lamp (a deflection of 100 divisions), as was done in the pressure observations. After shutting off the light the heating current was turned on. It was regulated by means of the variable resistance R_1 (Fig. 1), so that nearly the same throw was obtained from the galvanometer G_1 as when the bolometer was exposed to the lamp. All deflections of the galvanometer G_1 were taken with the

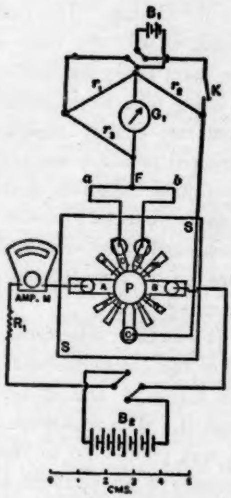


FIGURE 1.

bridge current both direct and reversed to eliminate any local disturbances in the bridge and also with the heating current both direct and reversed. A Siemens & Halske direct-reading precision milliampèremeter was used to measure this current. From repeated observations the current which gave the same heating effect as the light beam was $i = 0.865$ amp. The resistance between the binding posts A and B was measured with the lamp on and gave $r = 0.278$ ohm. The intensity of the beam in erg-seconds was thus: $r\, i^2 \times 10^7$, or $0.278 \times 0.75 \times 10^7$.

Using 0.92 as the reflection coefficient of silver, the pressure computed from the energy measurements was $p = 1.34 \times 10^{-4}$ dynes. The observed pressure was only 78 per cent of this value. No correction for the diffuse reflection of the blackened bolometer face nor for the difference in reflecting power between the two faces of the silver coating, discovered later, was made. The two corrections, however, nearly balance, so no considerable change in the result would have been effected by using them.

It was later discovered that, in dissolving the silver from the platinum when the bolometer was made, the acid had eaten away the silver from the strips A and B for a distance of nearly a millimeter under the asphalt. The resistance 0.278 ohm given for the disc was thus too high. It was impossible to redetermine the resistance by the direct method because of an accident to the bolometer by which the disc was nearly severed from the strip B. The disc was therefore carefully torn away from its supports, mounted on a glass plate and cut on a dividing engine into strips, 1 and 2 mms. wide, parallel to $A\,B$. The resistance along these strips was measured by the fall of potential method. The resistance was found to vary slightly in different parts of the disc due to lack of uniformity in the thickness of the metal. After many measurements, an average value was reached and the resistance of the disc computed theoretically as follows: —

The resistance of a conducting sheet of infinite extent, when the current enters and leaves the sheet by electrodes * of relatively great conductivity, is $\frac{\sigma}{4\pi C}$, where σ is the resistance of any square of the sheet, and C is the electrostatic capacity of the two electrodes. If the electrodes are cylinders, the lines of flow are circles orthogonal to them. When the sheet, in place of being infinite, is bounded by one of these circular lines of flow, the resistance is $\frac{\sigma}{2\pi C}$. In particular, if the elec-

* J. J. Thomson, Electricity and Magnetism, 2d Edition, p. 314. Cambridge, 1897.

trodes of radii r are on a diameter of this circular sheet of radius R, then the resistance can be shown to be

$$\frac{\sigma}{\pi} \log_e \frac{2r^2 + R^2 + R\sqrt{4r^2 + R^2}}{2r^2}.$$

Assuming for the moment that the leading-in strips of the bolometer (Fig. 1) were of great conductivity compared to that of the thin platinum sheet and that they terminated in circular arcs orthogonal to this circular sheet, the resistance would be $0.922 \times \sigma$, giving to r the value of 2.79 mms. and to R the value 11.25 mms. But the leading-in strips terminated on the boundary of the large circle. The resistance was therefore altered by two facts, — the lines of flow were changed and the distance between the electrodes was increased. The latter is the important item. It is necessary therefore to find approximately the resistance of these gibbous portions of the large disc previously considered as electrodes. This may be done by estimating the area of these parts and by considering the average equipotential line as midway between the chord and arc of the cylindrical electrode. It results that the amount to be added on account of this calculation is $0.471 \times \sigma$. Hence the resistance between the electrodes is now $(0.922 + 0.471)\ \sigma = 1.393 \times \sigma$. The value of σ as found by the fall of potential method was 0.148 at 19° C. When corrected for the temperature of the disc exposed to the lamp, σ becomes 0.160. Hence the resistance of the disc when hot was $1.393 \times 0.160 = 0.221$ ohm.* Substituting this computed value of the resistance in place of the one used, the energy of the standard beam becomes $0.221 \times 0.75 \times 10^7$ ergs-seconds and

$$p = \frac{1.92 \times 0.221 \times 0.75 \times 10^7}{3 \times 10^{10}} = 1.05 \times 10^{-4} \text{ dynes.}$$

This result is in accidental agreement with the observed pressure. If necessary corrections, determined by later experiment, had been applied, the difference between the observed pressure and the pressure computed from the energy measurements would have been about three per cent. Moreover the probable error of the final result was roughly double this amount.

In the November number of the Annalen der Physik for 1901 Professor Lebedew † published the results of a more varied series of

* The resistance of a trial disc was measured experimentally with the result that the experimental value differed from the theoretical by about one per cent.

† P. Lebedew, Ann. Phys., VI. 433 (1901).

measurements of radiation pressure than the early measurements of the present writers. The principal difference between the methods employed by him and by the writers for determining the pressure was that he used very thin metallic vanes surrounded by gas at extremely low pressures, thus following Maxwell's suggestion literally, while the writers used silvered glass vanes and worked at large gas pressures for which the gas action had been carefully and exhaustively studied and found to be negligibly small for short exposures. From our knowledge of the variation of gas action in different vacua, we feel sure that our method would not have been successful in high vacua because of the relatively large gas action. Professor Lebedew's own results, with blackened vanes of lower heat conductivity, show that his success in eliminating gas disturbance was due to the high heat conductivity of thin vanes rather than to the high vacua employed.

Professor Lebedew's * estimate of the accuracy of his work is such as to admit of possible errors of twenty per cent in his final results. An analysis of Professor Lebedew's paper and comparison with our preliminary experiments seems to show that his accidental errors were larger than ours, but through the undiscovered false resistance in the bolometer our final results were somewhat further from the theory than his. Either of the above researches would have been sufficient to establish the *existence* of a pressure due to radiation, but neither research offered, in our judgment, a satisfactory *quantitative* confirmation of the Maxwell-Bartoli theory.

Later Pressure Measurements.

Description of Apparatus. — The Torsion Balance.

The form of suspension of the torsion balance, used to measure radiation pressure in the present study, is seen in Fig. 2. The rotation axis *a b* was a fine rod of drawn glass. A drawn glass cross-arm *c*, bent down at either end into a small hook, was attached to the axis. The surfaces *C* and *D*, which received the light beam, were circular microscope cover-glasses, 12.8 mm. in diameter and 0.17 mm. thick, weighing approximately 51 mgs. each. To distinguish the two vanes from each other, in case individual differences should appear in the measurements, and also to mark the two faces of each vane for subsequent recognition, a letter *C* was marked on one, and *D* on the other by diamond scratches. Through each glass, a hole 0.5 mm. or less in diameter, was drilled near the edge,

* P. Lebedew, Ann. Phys., VI. 457 (1901).

by means of which the glasses could be hung on the hooks on the cross-arm *c*. On opposite sides of the rotation axis at *d* two other drawn-glass cross-arms were attached. The cover glassses slipped easily between these, and were thus held securely in one plane. Further down on *a b*, a small silvered plane mirror m_1 was made fast at right

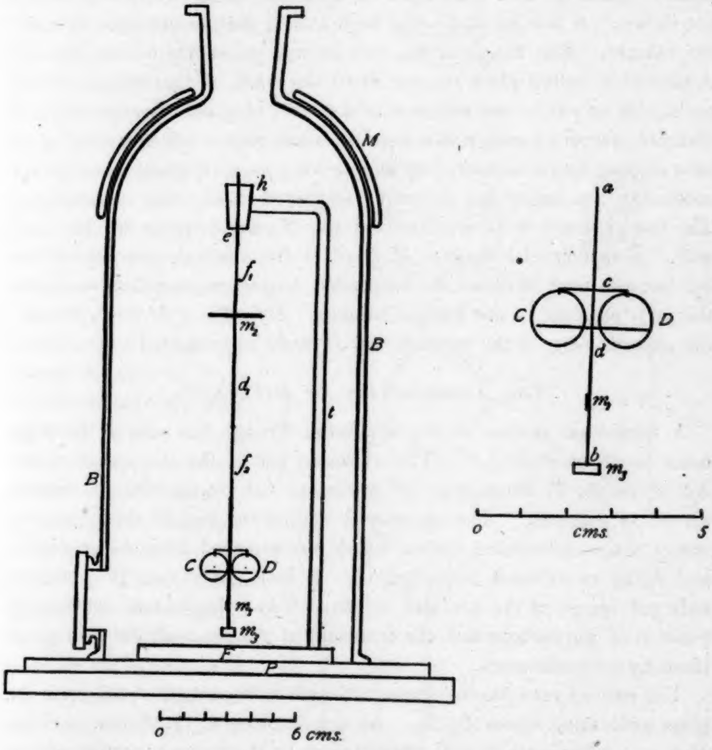


FIGURE 2.

angles to the plane of *C* and *D*. This mirror was polished bright on the silver side, so that the scale at S_5 (Fig. 3) could be read in either face. A small brass weight m_3 (Fig. 2), of 452 mgs. mass and of known dimensions, was attached at the lower end of *a b*. The cover-glasses which served as vanes were silvered and brilliantly polished on the silvered sides, and so hung on the small hooks that both silver faces or both glass faces were presented to the light. A quartz fiber f_2, 3 cms. long, was

made fast to the upper end of *a b*, and to the lower end of a fine glass rod d_1 which carried a horizontal magnet m_2. The rod d_1 was in turn suspended by a short fiber to a steel pin *e*, which could be raised or lowered in the bearing *h*. The whole was carried by a bent glass tube *t*, firmly fastened to a solid brass foot *F*, resting on a plane ground-glass plate *P*, cemented to a brass platform mounted on three levelling screws not shown. A bell-jar *B*, 25 cms. high and 11 cms. in diameter, covered the balance. The flange of the bell-jar was ground to fit the plate *P*. A ground-in hollow glass stopper fitted the neck of the bell-jar, which could thus be put in connection with a system of glass tubes leading to a Geissler mercury pump, a MacLeod pressure gauge, and a vertical glass tube dipping into a mercury cup and serving as a rough manometer for measuring the larger gas pressures employed during the observations. The low pressures were measured on the MacLeod gauge in the usual way. A semicircular magnet *M*, fitted to the vertical curvature of the bell-jar, was used to direct the suspended magnet m_2 and thus to control the zero position of the torsion balance. By turning *M* through 180°, the opposite faces of the vanes *C* and *D* could be presented to the light.

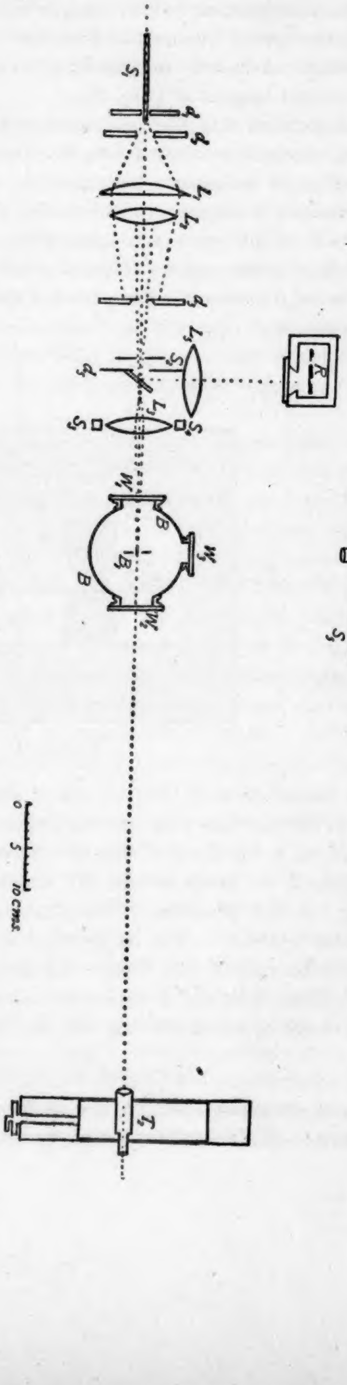
The Arrangement of Apparatus.

A horizontal section of the apparatus through the axis of the light beam is shown in Fig. 3. The white-hot end of the horizontal carbon S_1, of an A. T. Thompson 90° arc-lamp, fed by alternating current, served as a source. The arc played against the end of the horizontal carbon from the vertical carbon which was screened from the lenses L_1 and L_2 by an asbestos diaphragm d_2. A lens, not shown, projected an enlarged image of the arc and carbons on an adjacent wall, so that the position of the carbons and the condition of the arc could be seen at all times by both observers.

The cone of rays passing through the small diaphragm d_2 fell upon the glass condensing lenses L_1, L_2. At d_3 a diaphragm, 11.25 mm. in diameter, was interposed, which permitted only the central portion of the cone of rays to pass. Just beyond d_3, the beam passed to a shutter at S_2. This shutter was worked by a magnetic escapement, operated by the seconds contact of a standard clock. The observer at T_1 might choose the second for opening or closing the shutter, but the shutter's motion always took place at the time of the seconds contact in the clock. Any exposure was thus of some whole number of seconds' duration. The opening in the shutter was such as to let through, at the time of exposure, all of the direct beam which passed through d_3, but to shut out

stray light. Just beyond the shutter and attached to the diaphragm d_5 was a 45° glass plate which reflected a part of the beam to the lens L_5, by means of which an image of d_3 was projected upon one arm of a bolometer at R. The glass lens L_3 focused a sharp image of the aperture d_3 in the plane of the vanes of the torsion balance B_1 under the bell-jar. The bell-jar was provided with three plate glass windows W_1, W_2, W_3. The first two gave a circular opening 42 mm. in diameter, and through the third, deflections of the balance were read by a telescope and scale. The lens L_3 was arranged to move horizontally between the stops S_3 and S_4. These were so adjusted that when the lens was against S_3 the sharp image of the aperture d_3 fell centrally upon one vane; and when against S_4 the image fell centrally upon the other. This adjustment, which was a very important one, was made by the aid of a telescope T_2, mounted on the carriage of a dividing engine. This was used to observe and measure the position

FIGURE 3.



of the rotation axis, as well as the positions of the images of d_3, when the lens L_3 was against the stops. For the latter measurements, the vanes could be moved out of the way by turning the suspension through 90° by the control magnet M (Fig. 2).

To make sure that the balance as used was entirely free from any magnetic moment or disturbance, the small magnet m_2 was clamped in one position to maintain a constant zero, and the period of the balance was accurately measured with the axis of the large magnet M in the vertical plane of the vanes and again when the axis was at right angles to the plane of the vanes. Several series of this sort failed to show a difference of 0.1 second in the period of the balance for the two positions of the magnet.

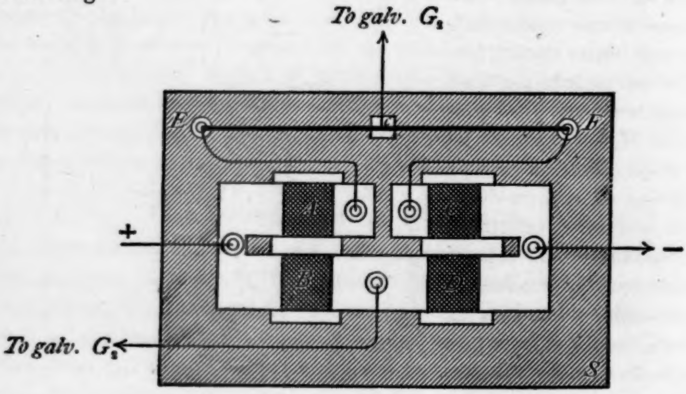


FIGURE 4.

The bolometer at R (Fig. 3) was of sheet platinum 0.001 mm. thick, rolled in silver. The strip was cut out in the form shown in Fig. 4, and mounted on a thin sheet of slate S. Two windows had been cut in the slate behind the strips at $ABCD$ where the silver had been removed leaving the thin platinum. The platinum surfaces were blackened by Kurlbaum's process. The image from L_5 (Fig. 3), fell at D. The silver ends between A and C were connected with E and F respectively. On the heavy wire EF a sliding contact c served to balance the bridge, all four arms of which are shown in the figure.

METHODS OF OBSERVATION.

The observations leading to the results given later were of three different kinds: (1) The calibration of the torsion balance; (2) the measure-

ment of the pressure of radiation in terms of the constant of the balance; and (3) the measurement of the energy of the same beam in erg-seconds by the rate of temperature rise of a blackened silver disc, of known mass and specific heat.

1. The determination of the constant of the torsion balance was made by removing the vanes C and D and accurately measuring the period of vibration. Its moment of inertia was easily computed from the masses and distribution of the various parts about the axis of rotation. The moment of torsion for 1 mm. deflection on a scale 105 cm. distant was 0.363×10^{-5} dyne $\times$ cm. This value divided by one-half the distance between the centres of the light spots on the two vanes gave the force in dynes per scale division deflection. As the light spots were circles 11.25 mm. in diameter the area of the image was very nearly 1 $(\text{cm.})^2$, hence the above procedure gave roughly the pressure in dynes per square centimeter.

2. In the measurements of radiation pressure, it was easier to refer the intensity of the beam at each exposure to some arbitrary standard which could be kept constant than to try to hold the lamp as steady as would otherwise have been necessary. For this purpose, the bolometer at R (Fig. 3) was introduced, and simultaneous observations were made of the relative intensity of the reflected beam by the deflection of the galvanometer G_2, and the pressure due to the transmitted beam by the deflection of the torsion balance. The actual deflection of the balance was then reduced to a deflection corresponding to a galvanometer deflection of 100 scale divisions. The galvanometer sensitiveness was carefully tested at the beginning and end of each evening's work. All observations of pressure were thus reduced to the pressure due to a beam of fixed intensity.

At each series of radiation pressure measurements, two sets of observations were made. In one of these sets, static conditions were observed, and in the other, the deflections of the balance due to short exposures were measured. In the static observations, each vane of the balance was exposed in turn to the beam from the lamp, the exposures lasting until the turning points of the swings showed that stationary conditions had been reached. The moment of pressure of radiation and gas action combined would thus be equal to the product of the static deflection and the constant of the balance. The torsion system was then turned through 180° by rotating the outside magnet, and similar observations were made on the reverse side of the vanes. All turning points of the swinging balance in these observations were recorded. From the data

thus obtained the resultant of the combined radiation and gas forces could be determined for the time of every turning point. Every value was divided by the deflection at standard sensitiveness of the galvanometer G_2 read at the same time and was thus reduced to a standard lamp. Results thus obtained, together with the ballistic measurements, showed the direction and extent of the gas action as well as its variation with length of exposure.

The reasons for reversing the suspension follow: The beam from the lamp, before reaching the balance, passed through three thick glass lenses and two glass plates. All wave-lengths destructively absorbed by the glass were thus sifted out of the beam by the time it reached the balance vanes. The silver coatings on the vanes absorbed therefore more than the glass. The radiation pressure was always away from the source irrespective of the way the vanes were turned, while the gas action would be exerted mainly on the silvered sides of the vanes.

At the close of the pressure and energy measurements, when the reflecting power of the silver faces of the vanes was compared with that of the glass-silver faces, the reflection from the silver faces was found very much higher than that for the glass faces backed by silver. This result was the more surprising because the absorption of the unsilvered vanes was found by measurement to be negligibly small.* This unexpected difference in reflecting power of the two faces of the mirrors prevented the elimination of the gas action, by the method described, from being as complete as had been hoped for. But by choosing a gas pressure where the gas action after long exposure is small, the whole gas effect during the time of a ballistic exposure may be so reduced as to be of little consequence in any case.

By exposing each of the vanes in turn and by reversing the suspension and averaging results, nearly all errors due to lack of symmetry in the balance or in the position of the light images with reference to the rotation axis, or errors due to lack of uniformity in the distribution of intensity in different parts of the image, could be eliminated.

The changing character of the gas action, both with time of exposure and gas pressure surrounding the balance vanes, is well illustrated in eight series of static observations in which the glass faces of both vanes were exposed.† The results obtained on the two vanes were averaged

* Lord Rayleigh records a similar difference between the reflection from air-silver and glass-silver surfaces. Scientific Papers, Cambridge, II. 538–539 (1900).

† Observations were also made on the silver faces, but the gas action when the glass faces were exposed was nearly double that for the silver faces, so the least favorable case is shown.

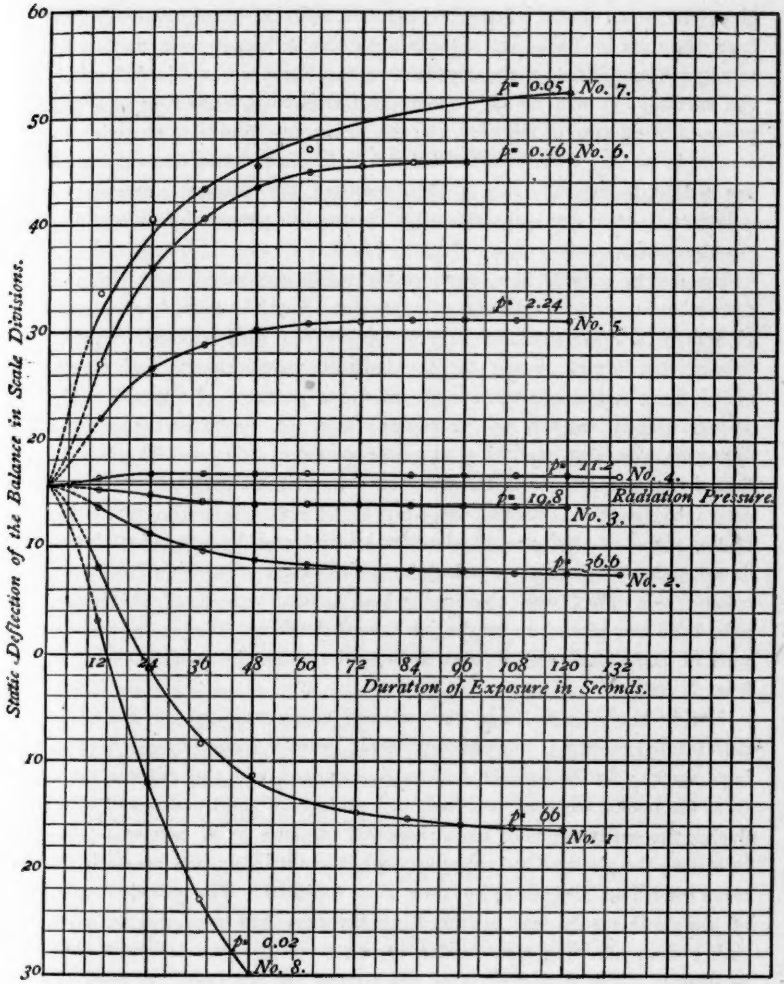


Figure 5.

and plotted as curves in Fig. 5, where static deflections due to combined radiation pressure and gas action are shown as ordinates and duration of exposure, in seconds, as abscissæ.* A horizontal line through the diagram

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* Ordinates of the curves are proportional to moments.

gives the mean value of the moment of radiation pressure computed from the data in Table II. Decrease of the deflection with time indicates gas repulsion on the warmed silver faces and increase in deflection, gas suction. It will be seen from the curves that beginning at a gas pressure of 66 mm. of mercury, the gas action was repulsion changing to suction in passing from 19.8 to 11.2 mm. In the last two cases the total gas action is small. For lower pressures the suction increases to 0.05 mm. At a gas pressure of 0.02 mm. the gas action is again a strong repulsion.

The curves indicate the existence of two gas pressures, at which the gas action in our arrangement of apparatus should be zero, one between 19.8 and 11.2 mm. and the other between 0.05 and 0.02 mm.* The former region was chosen for the ballistic measurements and nearly all of the observations were made at a gas pressure of approximately 16 mm. Even for the two pressures where the decrease in the static deflection was most rapid, i. e. at gas pressures of 66 and 0.02 mm., the first throw was always in the direction of radiation pressure. The gas action is strongly influenced by very slight changes in the inclination of the plane of the vanes to the vertical and also by any object introduced under the bell-jar anywhere near the vanes. For instance, a very considerable effect was observed when a small vessel of phosphoric anhydride was placed under the jar behind the vanes, though the nearest wall of the vessel was separated from the vanes by a distance of at least 3 cms.

During the observations, the polished silver coatings on the vanes deteriorated rapidly; new coatings rarely lasted for more than two evenings' work. As the balance had to be removed and the mirrors taken from the hooks, silvered, polished, and replaced a great number of times during the entire series of measurements, although great care was taken in setting the plane of the vanes vertical, it is not likely that precisely the same conditions for gas action were ever repeated. The principal value of the static results was in indicating favorable gas pressures for work, rather than affording quantitative estimates of the gas action in short exposures. The dotted parts of the curves are not based on results of observation and might perhaps have been omitted without loss.

* Crookes in his work with the radiometer discovered certain gas pressures for which the combined gas and radiation forces neutralized, but as he did not discriminate between forces due to radiation and gas forces his results were apparently capricious and his reasoning somewhat confused. See Phil. Trans., p. 519 (1875).

It was plain, therefore, that further elimination of the gas action must be sought in exposures so short that the gas action would not have time to reach more than a small fraction of its stationary value. This led to the method of ballistic observations.

The Ballistic Observations.

In passing from the static to the ballistic observations it must always be possible to compute the static equivalent of the ballistic swings. Furthermore the exposures should be made as short as possible without reducing the size of the swing below a value which can be accurately measured.

If the exposure lasts for one-half the period of the balance, the deflection, if the gas action be small and the damping zero, is equal to 2θ, where θ is the angle at which the torsion of the fibre will balance the moment produced by the radiation pressure. If the duration of the exposure be one-quarter of the period of the balance, the angle of deflection is $\theta\sqrt{2}$. The deflection is thus reduced by 30 per cent, but the effect of the gas action is reduced in greater proportion. It was decided therefore to expose for six seconds, one-quarter of the balance period. Neglecting the gas action, the equation * of motion of the balance is given by

$$\kappa \frac{\partial^2 \theta}{\partial t^2} + 2\epsilon \frac{\partial \theta}{\partial t} = - G\theta + L$$

where κ = the moment of inertia of the torsion balance,
ϵ = the damping constant,
G = the moment of torsion of the fibre for $\theta = 1$ radian,
and L = the moment of the radiation force.

The solution of this equation is

$$\theta = \frac{L}{G}\left\{1 - e^{-\frac{\epsilon}{\kappa}t} \cos \sqrt{\frac{G}{\kappa} - \frac{\epsilon^2}{\kappa^2}}\, t\right\}$$

$$= \frac{L}{G}\left\{1 - e^{-\frac{\epsilon}{\kappa}t} \cos 2\pi \frac{t}{T}\right\} \qquad (1)$$

the constants of integration having been determined from the condition that

$$\theta = \frac{\partial \theta}{\partial t} = 0 \text{ when } t = 0.$$

* We are justified in using quantitatively this equation, containing a damping term proportional to the velocity, because the amplitudes of the successive swings of the torsion balance, when no energy fell upon the vanes, were found experimentally to follow accurately the exponential law.

When

$$t=\frac{T}{4},\ \theta=\frac{L}{G}\text{ and }\frac{\partial\theta}{\partial t}=\frac{L}{G}\left(\frac{\epsilon}{\kappa}e^{-\frac{\epsilon}{\kappa}t}\cos 2\pi\frac{t}{T}+e^{-\frac{\epsilon}{\kappa}t}\frac{2\pi}{T}\sin 2\pi\frac{t}{T}\right) \quad (2)$$

The light being cut off when $t=\frac{T}{4}$, the equation of motion becomes

$$\kappa\frac{\partial^2\theta}{\partial t^2}+2\epsilon\frac{\partial\theta}{\partial t}=-G\theta \quad (3)$$

the solution of which is $\theta=A e^{-\frac{\epsilon}{\kappa}t}\cos\left(2\pi\frac{t}{T}+\alpha\right)$ where A and α can be determined by the conditions imposed by equation (2). Neglecting very small quantities, the value of the amplitude A is expressed by the equation

$$A=\frac{L}{G}\left\{1+r+\frac{2}{\pi}r^{\frac{1}{2}}\log\left(\frac{1}{r}\right)\right\}^{\frac{1}{2}}, \quad (4)$$

where r is the ratio of successive amplitudes of the damped vibrations. If $r=1$, that is if the motion is undamped, $A=\frac{L}{G}\sqrt{2}$. In the partial vacuum used in the experiments (16 mms. of mercury, a value chosen from the curves in Fig. 5), r was found to be equal to 0.783; consequently $A=1.357\,\frac{L}{G}$. (5)

From this it is seen that the total angle of deflection of the torsion balance in the ballistic measurements is equal to 1.357 times the angle at which the moment of the torsion of the fibre balances the moment of the radiation pressure.

The duration of exposure was always six seconds without appreciable error, but the period of the balance on account of slight accidental shifting of small additional masses upon the counterpoise weight m_3 (Fig. 2), differed from twenty-four seconds sometimes by one per cent. It is necessary therefore to find the error in the deflection due to this variation in the period. This is done by making $t=\frac{T}{4}+\delta$ in equation (2) and in introducing the new conditions in equation (3). But it is simpler and sufficiently accurate to assume the motion as undamped. For this condition, the amplitude

$$A=\frac{L}{G}\left\{2+2\sin 2\pi\frac{\delta}{T}\right\}^{\frac{1}{2}}=\sqrt{2}\,\frac{L}{G}\left(1+\pi\frac{\delta}{T}\right)\text{ nearly.}$$

For $T = 23.75$ seconds $\frac{T}{4} = 5.94$ and $\delta = 0.06$. Hence

$$A = \sqrt{2}\frac{L}{G}(1.008).$$

If $\delta = 0$, $A = \sqrt{2}\frac{L}{G}$, consequently an error of 1 per cent in T causes an error of 0.8 per cent in A.

To make sure that the observed radiation pressures depended only on the intensity of the beam, and were uninfluenced by the wave length of the incident energy, the ballistic observations of pressure, the thermal measurements of intensity, and the determination of the reflection coefficients, were carried out for three entirely different wave-groups of the incident radiation. In the measurements designated "through air," no absorbing medium was introduced in the path of the beam between the lamp and the balance except the glass lenses and plates already mentioned. In the measurements "through red glass," a plate of ruby glass was put in the path of the beam between L_2 and d_3 (Fig. 3). For the observations "through water cell," a 9 mm. layer of distilled water in a glass cell was placed in the path of the beam at the same point.

The separate observations entering into a single series of ballistic measurements and their treatment will appear from Table II, which is copied direct from the laboratory notebook and represents an average ballistic series. The designations EVC_s, WVD_s, EVD_g, and WVC_g mean that the vane C in the first case was on the east side of the rotation axis with its silver face toward the light. The subscript g signifies that the glass face of the vane was toward the light. The second column of the table gives the zero reading of the balance before opening the shutter; the third, the end of the swing produced by a six-second exposure; the fourth, the deflection of the balance; the fifth, the ballistic deflection of the lamp galvanometer G_2. Columns six and seven give the balance deflection reduced to standard lamp.

The results of all the ballistic pressure measurements "through air" are collected in Table III. In the fourth and fifth columns two values are given for the constant of the lamp galvanometer G_2; since reversing the magnet on the balance bell-jar to reverse the suspension within affected the constant of the galvanometer slightly the values for the silver and glass faces forward were never the same. The subscripts show to which series, silver or glass, the constant belongs. The values of the lever-arm l of the balance, in the sixth column, are obtained by mea-

TABLE II.

AUGUST 28. LIGHT PRESSURE. BALLISTIC MEASUREMENTS. AIR.

Surface.	Zero.	Throw	Deflection.	Lamp.	Deflection (Lamp 100) E. V.	Deflection (Lamp 100) W. V.
			mms.	mms.	mms.	mms.
E V C_s	281.4	248.5	32.9	164.3	20.0	
W V D_s	281.5	313.9	32.4	164.5		19.7
E V C_s	281.4	249.8	31.6	157.9	20.0	
W V D_s	281.5	310.5	29.0	147.0		19.8
E V C_s	281.5	252.6	28.9	144.8	20.0	
W V D_s	281.5	309.6	28.1	141.8		19.8
E V C_s	281.5	252.9	28.6	143.5	19.9	
W V D_s	281.5	309.3	27.8	140.4		19.8
				Average . .	19.97	19.77
				Average, $\frac{C_s + D_s}{2} = 19.87$		
		Magnet reversed.				
E V D_g	280.1	246.0	34.1	180.4	18.92	
W V C_g	280.0	317.8	37.8	187.3		20.20
E V D_g	279.8	247.2	32.6	170.8	19.20	
W V C_g	279.4	313.7	34.3	169.4		20.25
E V D_g	279.1	248.9	30.2	161.1	18.80	
W V C_g	279.0	311.9	32.9	161.6		20.35
E V D_g	279.0	249.1	30.0	158.9	18.90	
W V C_g	278.9	311.2	33.2	164.4		20 20
				Average . .	18.97	20.25
				Average, $\frac{C_g + D_g}{2} = 19.61$		

suring the distance between the centres of the images when on the east and west vanes (by the dividing engine T_2, Fig. 3) and dividing by two. The columns headed $\frac{C_s + D_s}{2} = P_s$ and $\frac{C_g + D_g}{2} = P_g$ are the average

TABLE III.— RADIATION PRESSURE. BALLISTIC MEASUREMENTS, THROUGH AIR.

Date.	Air Pressure in mms. of Hg.	Balance Period T.	Sens. of Galvan'r.		l = Lever Arm. cms.	$\frac{C_s+D_s}{2}=P_s$.	$\frac{C_g+D_g}{2}=P_g$.	P_s corrected for $T=24''$.	P_g corrected for $T=24''$.	$\frac{P_s \times G_s}{l}$.	$\frac{P_g \times G_g}{l}$.	Average.
			G_s (Silver).	G_g (Glass.)								
June 19	32.5	23.75	[734]		.814	Average = 18.88		Average = 18.73		Average = 16.89		16.89
" 20	32.5	23.75	756	768	.814	19.67	16.94	19.51	16.81	18.12	15.86	17.00
" 23	37.0	23.75	700	716	.814							
July 23	16.0	23.75	682	707	.831	21.16	20.42	21.00	20.26	17.25	17.25	17.25
" 25	16.6	23.75	684	684	.815							
" 26	16.6	23.75	720	710	.815	19.34	19.98	19.18	19.82	16.93	17.28	17.10
Aug. 27	16.8	23.82	724	710	.823	20.16	19.40	20.00	19.25	17.60	16.61	17.10
" 28	15.7	23.82	721	716	.824	19.87	19.61	19.73	19.48	17.26	16.97	17.10
" 29	13.7	23.82	712	701	.824	19.68	20.07	19.53	19.92	16.90	16.97	16.94
" 31	14.0	24.00	718	713	.810	18.55	18.94	18.55	18.94	16.44	16.60	16.52
Sept. 1	16.6	24.00	692	672	.808	19.14	20.17	19.14	20.17	16.40	16.78	16.59
" 20	16.4	23.78	670	676	.812	20.96	20.02	20.81	19.87	17.17	16.54	16.86
" 23	16.4	23.78	666	684	.816	21.32	20.27	21.16	20.12	17.27	16.87	17.07
" 24	16.2	23.78	667	669	.816	20.76	19.80	20.60	19.65	16.84	16.11	16.47
								Average	. . . . .	17.11	16.71	16.91 ± 0.053

moments due to pressures for the silver and glass sides of the vanes respectively toward the light. The next two columns contain these moments corrected for a period of 24 seconds of the torsion balance. The columns headed $\frac{P_s \times G_s}{l}$ and $\frac{P_g \times G_g}{l}$ are the corresponding forces reduced to standard sensitiveness, $G = 1000$. The final column contains the averages of the two columns which precede it. Table IV exhibits corresponding data for "red glass" and "water cell." The air pressure, period of the balance, lever arm and galvanometer constants are those given in Table III for the same date.

In these ballistic measurements the lamp reading was the throw due to an exposure of the light upon the bolometer for six seconds, but in the energy measurements the lamp reading was a stationary deflection due to prolonged exposure. To bring the pressure values into comparison with the energy measurements it is necessary to reduce the average of the quantities in the last column to pressures in dynes by multiplying by 0.363×10^{-5}, the torsion coefficient of the quartz fibre, and to reduce not only to a static deflection of the torsion balance but also to a static deflection of the lamp galvanometer G_2. The ratio of a ballistic to a static deflection of the galvanometer G_2 was obtained from a long series of lamp exposures. This ratio was found "through air" to be $= 1.55$; "through red glass" $= 1.535$; "through water cell" $= 1.502$. These differences are probably due not solely to the damping constant of the galvanometer but to the peculiar manner in which the bolometer was warmed up to its stationary conditions by the beam from the lamp. Applying these reduction factors to the averages in Tables III and IV, we obtain the following results. The pressure of the standard light beam which has passed

$$(a)\ \text{through air} = 16.91 \times \frac{1.55}{1.357} \times 0.363 \times 10^{-5} = (7.01 \pm 0.023) \times 10^{-5}\ \text{dynes};$$

$$(b)\ \text{through red glass} = 16.91 \times \frac{1.535}{1.357} \times 0.363 \times 10^{-5} = (6.94 \pm 0.024) \times 10^{-5}\ \text{dynes};$$

$$(c)\ \text{through water cell} = 16.20 \times \frac{1.502}{1.357} \times 0.363 \times 10^{-5} = (6.52 \pm 0.028) \times 10^{-5}\ \text{dynes}.$$

TABLE IV.

Radiation Pressure. Ballistic Measurements.

Date.	$\frac{C_s + D_s}{2}$.	$\frac{C_g + D_g}{2}$.	P_s corrected for $T = 24''$.	P_g corrected for $T = 24''$.	$\frac{P_s \times G_s}{l}$.	$\frac{P_g \times G_g}{l}$.	P Average.
Through Water Cell.							
June 20	18.62	17.10	18.46	16.96	17.14	16.00	16.57
July 25	19.00	20.10	18.85	19.94	15.82	16.74	16.28
" 26	18.03	19.39	17.89	19.33	15.80	16.84	16.32
Aug. 27	18.63	18.66	18.50	18.53	16.29	15.99	16.14
" 29	18.25	19.02	18.10	18.87	15.68	16.06	15.87
Sept. 20	20.39	19.14	20.23	19.00	16.69	15.82	16.25
" 23	20.21	19.51	20.05	19.36	16.37	16.23	16.30
" 24	19.84	18.91	19.69	18.77	16.10	15.40	15.70
				Average . . .	16.24	16.15	16.20 ±0.066
Through Red Glass.							
June 23	19.99	18.40	19.83	18.26	17.05	16.06	16.56
July 25	20.70	20.94	20.54	20.77	17.24	17.43	17.33
Aug. 27	19.97	19.25	19.82	19.10	17.46	16.46	16.96
" 28	19.99	19.42	19.84	19.28	17.36	16.75	17.05
" 29	19.99	19.92	19.84	19.77	17.14	16.82	16.98
" 31	18.98	19.14	18.98	19.14	16.82	16.84	16.83
Sept. 20	21.00	19.97	20.83	19.82	17.19	16.50	16.84
" 23	21.48	20.34	21.31	20.18	17.39	16.92	17.15
" 24	21.00	19.68	20.83	19.53	17.03	16.03	16.53
				Average . . .	17.18	16.65	16.91 ±0.051

The Energy Measurements.

Before rejecting the bolometer method used in the preliminary measurements of energy, a second bolometer of slightly different construction was tried; but the lack of uniformity of resistance, already mentioned, made its indications too uncertain for the present work. The radiant intensity of the beam used in the later experiments was determined by directing it upon the blackened face of a silver disc, weighing 4.80 grams, of 13.3 mm. diameter and of 3.58 mm. thickness, and by measuring its rate of temperature rise as it passed through the temperature of its surroundings. The disc was obtained from Messrs. Tiffany & Co. and was said by them to be 99.8 per cent fine silver. Two holes were bored through parallel diameters of the disc, one-fourth of the thickness of the disc from either face. Two iron-constantan thermo-junctions, made by soldering 0.1 mm. wires of the two metals, were drawn through the holes into the centre of the disc. To insulate the wires from the disc, fine drawn glass tubes were slipped over them and thrust into the holes, leaving less than 2 mm. bare wire on either side of the junctions. The wires were sealed into the tubes, and the tubes into the disc by solid shellac. The tubes projected 15 mm. or more from the disc and were bent upward in planes parallel to the faces of

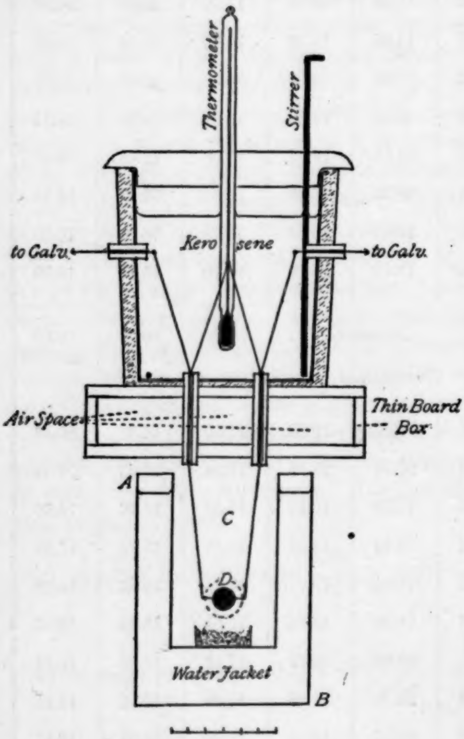

Figure 6.

the disc. The general arrangement will be seen in Fig. 6. The disc was suspended by the four wires some distance below a small flat wooden box. On the box was fastened a calorimeter can swathed in cotton and filled with kerosene in which the constant thermo-junctions were immersed. Copper wires soldered to the two ends of the thermo-electric series were brought out of the calorimeter, and the circuit was closed through 1000 ohms in series with the 500 ohms resistance of galvanometer G_1. The thermo-junctions in the disc were in series, and as each junction was midway between the central plane of the disc and either face, it was assumed that when the disc was slowly warmed by heating one face the electromotive forces obtained corresponded to the mean temperature of the disc. One face of the disc was blackened by spraying it with powdered lampblack in alcohol containing a trace of shellac. This method was suggested by Prof. G. E. Hale and gives very fine and uniform dead black coatings not inferior to good smoke deposits.

For the energy measurements the bell-jar and the torsion balance were removed from the platform P (Fig. 2) and a double walled copper vessel, AB (Fig. 6), which served as a water jacket surrounding a small air chamber C, was mounted in the same place. A tube 2 cm. in diameter was soldered into the front face of the jacket to admit the light beam into the chamber C. This opening was covered by a piece of plate glass similar to the plates forming the larger windows in the bell-jar.

The needle system in G_1, a four-coil du Bois Rubens galvanometer, was suspended in a strong magnetic field so that its period was about four seconds. The system was heavily damped by a mica air-fan of large surface. The disc junctions and galvanometer responded quickly to the radiation, as was shown by the reversal of motion of the magnet system 1.2 seconds after the light was cut off from the disc when the latter was a few degrees above the temperature of the room.

The disc was calibrated for temperature in terms of the deflection for a definite sensitiveness of the galvanometer G_1. For this purpose the disc was immersed in a kerosene bath and the galvanometer deflection measured for two different temperatures of the disc. One of these was about 18° C. above the comparatively steady temperature of the room, or calorimeter containing the standard temperature junctions (see Fig. 6), and the other about the same number of degrees below the room temperature. These two temperatures were measured by a Fuess Standard Thermometer divided into tenths of a degree and calibrated at the Reichsanstalt. Two calibrations of the silver disc were made some days apart. One of these series appears in full in Table V. The first three

TABLE V.

Calibration of Silver Disc.

Cold Bath.

G_1 Readings.			Disc T_1°.	Room cal. T_2°.	Deflection of G_1.	Means of alternate Deflec'ns.	G_1 Means.	T_2°–T_1°.
Rev.	Zero.	Direct.						
	221.2 *221.0*	35.2	1°.58	20°.05	185.8			
402.0	220.9 *221.0*		1°.60	20°.07	181.0	185.7	183.4	18.47
	221.2 *221.2*	35.7	1°.57	20°.10	185.5	181.3	183.4	18.53
403.1	221.2 *221.5*		1°.56	20°.14	181.6	185.8	183.7	18.58
	221.9 *222.0*	35.9	1°.54	20°.16	186.1	182.2	184.1	18.62
405.0	222.1 *222.2*		1°.52	20°20	182.8	186.3	184.5	18.68
	222.4 *222.7*	36.2	1°.57	20°.22	186.5	182.7	184.6	18.65
405.8	223.0 *223.1*		1°.60	20°.26	182.7	186.7	184.7	18.66
	223.2 *223.3*	36.3	1°.59	20°.30	187.0			
	223.5						184.0	18°.60

Correction to $T_1 = 0°.00$.

Warm Bath.

Rev.	Zero.	Direct.	Disc T_1°.	Room cal. T_2°.	Deflection of G_1.	Means of alternate Deflec'ns.	G_1 Means.	T_2°–T_1°.
	217.3 *218.6*	434.2	41°.45	20°.40	215.6			
2.0	219.9 *220.5*		41°.35	20°.42	218.5	213.7	216.1	20.93
	221.1 *222.4*	434.2	41°.25	20°.44	211.8	216.4	214.1	20.81
10.0	223.7 *224.4*		41°.08	20°.50	214.4	209.1	211.7	20.58
	225.1 *225.7*	432.1	40°.90	20°.55	206.4	212.4	209.4	20.35
16.3	226.3 *226.7*		40°.80	20°.60	210.4	204.8	207.6	20.20
	227.2 *227.8*	431.0	40°.67	20°.61	203.2	208.5	205.8	20.07
21.8	228.4 *228.5*		40°.55	20°.63	206.7	201.8	204.2	19.92
	228.7 *229.0*	429.4	40°.43	20°.65	200.4			
	229 3						209.8	20°.41

Correction to $T_1 = 0°.10$. Corrected, 20.51

columns of the table give the zero, direct and reversed reading of the galvanometer G_1. The fourth column gives the temperature of the bath in which the disc was immersed, and the fifth, that of the constant temperature calorimeter. The sixth column gives the deflections of G_1. The seventh column the means of the alternate deflections. The eighth, the mean of the two columns preceding it. The last column gives the difference in temperature between the two calorimeters in degrees C. For the total temperature range in the table, 39.11°, the deflection of G_1 was 393.8 scale divisions for a sensitiveness of $G_1 = 996$. A range of one degree would thus give a deflection of 10.03 divisions for a sensitiveness of $G_1 = 1000$. The mean of two separate calibrations was 9.96 scale divisions for one degree temperature difference.

Before beginning a series of intensity measurements the disc was suspended in an air-chamber containing phosphoric anhydride and surrounded by a jacket of ice and salt. The disc was thus lowered to a temperature of about zero degrees and was then quickly transferred to the chamber C (Fig. 6), and the beam was directed upon it. When its temperature had risen to within five or six degrees of that of the chamber C, galvanometer readings were made at intervals of five seconds until the disc was heated to a temperature several degrees above its surroundings. The temperature of the chamber C was determined by removing the disc and cooling it to a point near the room temperature, then replacing it and observing its rate of temperature change for several minutes.

The notebook record of one series of observations showing the heating of the disc by the light beam is given in full in Table VI. It will be seen from the table that the temperature of the disc passed that of the chamber thirty seconds after the beginning of the series. The readings of G_1 at equal time intervals on either side of the zero are on horizontal lines. The last column of the table contains the rate at which the galvanometer deflection was changing when the disc and its surroundings were at the same temperature.

Energy series were made "through air," "through red glass," and "through water cell," as in the pressure measurements. During the experiment the black coatings were frequently cleaned off from the disc and new ones deposited. The final result therefore does not correspond to an individual, but to an average coating.

To correct for any inequality between the two disc thermo-junctions or any lack of symmetry in their positions, referred to the central plane of the disc, which might prevent the mean temperature of the two junctions from representing the mean temperature of the mass, series of

TABLE VI.

August 16. Energy Measurements. Through Air. Series 4.

Zero of Galvanometer G_1 (closed circuit) determined by method of cooling $= 216.8 =$ reading at room temperature.

Time.	G_1.	Time.	G_1.	ΔG_1.	Δt.	$\frac{\Delta G_1}{\Delta t}$ (in mm. per sec.)
0 secs.	174.5	60 secs.	253.2	78.7	60 secs.	1.312
5 "	182.0	55 "	247.3	65.3	50 "	1.306
10 "	189.0	50 "	241.3	52.3	40 "	1.308
15 "	196.2	45 "	235.2	39.0	30 "	1.300
20 "	203.0	40 "	229.1	26.1	20 "	1.305
25 "	209.7	35 "	222.8	13.1	10 "	1.310
30 "	216.4			Average		1.307

The lamp reading (G_2) was 924.
The sensitiveness of G_2 was 667, and of G_1 was 996.
$\frac{\Delta G_1}{\Delta t}$ reduced to standard conditions becomes

$$1.307 \times 667 \times 996 \div (924 \times 1000) = 0.943 \text{ mm. per sec.}$$

observations were made on each face of the disc. The black coating was always cleaned off from the face of the disc away from the light. All of the series of energy measurements are gathered together in Tables VII and VIII. In the tables, under the head "through air," the first column contains the observed rate of increase in the galvanometer deflection G_1, when the disc and its surroundings were at the same temperature; the second column, the corresponding mean lamp deflections of galvanometer G_2. The third and fourth columns contain the sensitiveness of galvanometers G_1 and G_2 respectively, and the last column the values of the first column reduced to standard lamp and standard sensitiveness of both instruments. The series on the two faces of the disc are recorded and averaged separately, then combined with their probable errors in the general average at the end of Table VIII.

Tables VII and VIII give the following results. The average increase in the reading of G_1 for standard conditions is 0.966 mm. per second. From the thermal calibration, a deflection of 9.96 divisions corresponds

TABLE VII. — FRONT FACE.

Date.	Through Air. $\frac{\partial G_1}{\partial t}$.	Through Air. G_2 (Lamp).	Through Air. S_1.	Through Air. S_2.	Through Air. $\frac{\partial G_1}{\partial t}$ reduced to Standard.	Through Red Glass. $\frac{\partial G_1}{\partial t}$.	Through Red Glass. G_2.	Through Red Glass. $\frac{\partial G_1}{\partial t}$ reduced to Standard.	Through Water Cell. $\frac{\partial G_1}{\partial t}$.	Through Water Cell. G_2.	Through Water Cell. $\frac{\partial G_1}{\partial t}$ reduced.
Aug. 10	1.387	980	990	689	.965				.437	345	.864
" "	1.263	920	990	689	.936				.400	311	.877
" "									.369	279	.902
" 11	1.244	866	986	701	.992	.750	546	.950	.412	315	.905
" "	1.455	1010	986	701	.995	.750	546	.950	.510	382	.922
" "	1.505	1047	986	701	.994				.516	381	.935
" 16	1.447	1022	996	669	.942	.736	529	.927	.416	327	.873
" "	1.284	886	996	669	.966	.740	527	.936	.451	352	.853
" "	1.316	925	996	669	.948	.797	550	.965	.502	382	.875
" "	1.307	924	996	669	.943						
" 18	1.598	1110	995	667	.955	.738	515	.952	.449	333	.895
" "	1.550	1047	995	667	.984	.732	518	.940	.445	342	.865
" "	1.548	1081	995	667	.995	.730	518	.938	.451	346	.867
" "	1.410	957	995	667	.977						
" "	1.330	898	995	667	.983						
" 19	1.241	862	1001	675	.975	.760	532	.965	.451	343	.892
" "	1.360	934	1001	675	.985	.728	512	.960	.452	338	.904
" "	1.324	905	1001	675	.990	.738	525	.950	.466	351	.898
" "	1.364	934	1001	675	.988						
			Average . . .		0.973 ±0.003	Average . .		0.948 ±0.002	Average . .		0.888 ±0.004

TABLE VIII. — Rear Face.

Date.	Through Air.					Through Red Glass.			Through Water Cell.		
	$\frac{\partial G_1}{\partial t}$.	G_2 (Lamp).	S_1.	S_2.	$\frac{\partial G_1}{\partial t}$ reduced to Standard.	$\frac{\partial G_1}{\partial t}$	G_2.	$\frac{\partial G_1}{\partial t}$ reduced.	$\frac{\partial G_1}{\partial t}$.	G_2.	$\frac{\partial G_1}{\partial t}$ reduced.
Aug. 12	1.374	960	991	684	.970	.808	578	.949	.495	370	.906
" "	1.331	932	991	684	.968	.740	536	.935	.434	320	.919
" "	1.284	900	991	684	.967	.765	542	.957	.489	371	.895
" 15	1.428	992	996	670	.960	.703	506	.926	.490	368	.890
" "	1.428	984	996	670	.968	.742	526	.941	.466	352	.885
" "	1.531	1068	996	670	.962	.765	551	.926	.440	337	.873
" 20	1.477	1047	996	685	.961	.703	522	.918	.458	375	.833
" "	1.520	1090	996	685	.951	.760	537	.965	.497	400	.848
" "	1.576	1130	996	685	.951	.781	570	.935	.507	408	.848
" "	1.568	1124	996	685	.950						
" 21	1.783	1224	995	668	.970	.846	604	.932	.503	393	.852
" "	1.773	1232	995	668	.957	.790	575	.915	.481	377	.850
" "	1.705	1190	995	668	.953	.803	575	.930	.483	373	.862
" "	1.452	1019	995	668	.948						
			Average . . .		0.960 ± 0.0014	Average . .		0.936 ± 0.003	Average . .		0.872 ± 0.005
	Average of front and rear face,				0.966 ± 0.0034			0.942 ± 0.0036			0.880 ± 0.0064

to a temperature difference of 1° C. Consequently the rise in temperature of the silver disc per second when the light passed: —

(*a*) through air $= 0.966 \div 9.96 = (0^\circ.0970 \pm 0^\circ.00034)$ C.;

(*b*) through red glass $= 0.942 \div 9.96 = (0^\circ.0946 \pm 0^\circ.00036)$ C.;

(*c*) through water cell $= 0.880 \div 9.96 = (0^\circ.0884 \pm 0^\circ.00064)$ C.

The mass of the silver disc was 4.80 grams, its specific heat* at 18° C. = 0.0556; the mechanical equivalent of heat at 18° C. $= 4.272 \times 10^7$ ergs.† Consequently the energy of the standard radiation is

(*a*) through air, $0.0970 \times 4.80 \times 0.0556 \times 4.272 \times 10^7$

or $E_a = (1.108 \pm 0.004) \times 10^6$ ergs per second.

(*b*) through red glass, $E_g = (1.078 \pm 0.004) \times 10^6$ " " "

(*c*) through water cell, $E_w = (1.008 \pm 0.007) \times 10^6$ " " "

Reflecting Power of the Surfaces Used.

According to Maxwell, the pressure in dynes per square centimeter for normal incidence is equal to the energy in ergs in unit volume of the medium. The energy in unit volume is made up of both the direct and reflected beams. If E is the intensity of the incident beam and ρ the reflection coefficient, the pressure $p = \frac{E(1+\rho)}{V}$, where V is the velocity of light. The methods for measuring p and E have already been described. The determination of ρ for both sides of the vanes C and D was made as follows. The supports of the torsion balance were replaced by the divided circular plate A (Fig. 7), of a force table which could be rotated about a central, vertical axis. The rod about which the plate turned passed up through the plate and at its top the mirror holder $b\,b$ was fastened. The vanes were freshly silvered and mounted on a plate-glass carrier $a\,a$, which was held by a clamp against the back face of $b\,b$. The beam was directed on the vanes by the lens L_3 (Figs. 3 and 7) exactly as it had been in the pressure observations. After reflection from the vane the beam fell on a concave mirror M which projected an image of the vane upon a simple sheet bolometer B, forming the unknown resistance of a postoffice-box bridge. The current was supplied from storage cells and the galvanometer was the same used in the energy determinations but fitted with low resistance coils. The bolometer was covered by the bell-jar used earlier. The mirror M, the bell-jar and bolometer were attached to the plate of the force table. The full line

* U. Behn, Ann Phys., IV. 266 (1900).

† Mean of Rowland's and Griffith's values, Phil. Trans., V. 184, 496 (1893).

diagram shows the arrangement for reflection. The dotted figure shows the position for a measurement of the direct beam. All measurements of direct reflection were made for an angle of incidence of 12°.5.

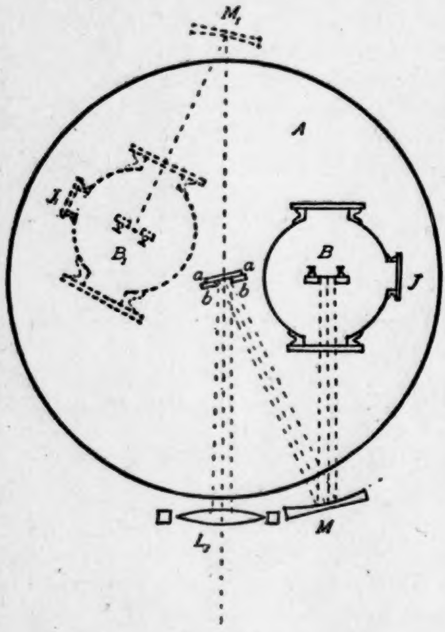

FIGURE 7.

The method of observing will be seen from the notebook record of a single series of measurements given in Table IX. In the table, D and R indicate direct and reflected beams, respectively. The first and second columns contain the zero points and end of swings of the galvanometer G_1, and the third column, the deflection. The remaining columns, in order, contain the lamp galvanometer deflection; the deflection of G_1 reduced to constant lamp; the means of each pair of D or R values; the means of alternate readings; and the final column, the quotients of the two preceding columns which are the reflection coefficients sought. In all, three series of measurements were made on the silver, and two series on the glass-silver faces of each vane. To get average coefficients which would represent the range of condition of the mirrors during the pressure measurements, the vanes were cleaned and new silver coatings deposited between each two series on the same vane. The reflection coefficients are collected in Table X. For each surface studied the diffused reflection for a beam which had traversed air was determined by setting the mirror holder for normal incidence. The diffuse energy reflected at an angle of 25° falling on the full aperture of the mirror M was measured, and the total diffuse energy for the hemisphere computed on the basis of the cosine law. If $I_0\, \partial A$ is the amount of diffuse radiation falling normally upon the area ∂A, distant r from the vane and at

TABLE IX.

OCTOBER 31, 1902. REFLECTION COEFFICIENT OF D_g. AIR.

G_1 Zero.	G_1 Turning Point.	Deflect. G_1	Lamp.	G_1 reduced to Standard	Averages.	Alternate Averages.	Reflect'n Coeffi't.
R 350.0	159.5	190.5	132.6	143.8	142.5		
349.0	152.0	197.0	139.3	141.3			
D 349.5	100.5	249.0	136.8	182.1	182.5	142.0	.779
350.0	111.5	238.5	130.5	183.0			
R 346.0	177.0	169.0	119.3	141.2	141.5	182.5	.775
347.0	171.0	176.0	124.4	141.7			
D 348.5	123.0	225.5	124.0	181.8	182.4	141.1	.773
348.5	120.0	228.5	125.0	183.0			
R 345.0	172.0	173.0	122.6	141.0	140.6	182.7	.770
345.0	171.0	174.0	124.0	140.3			
D 346.0	132.0	214.0	115.5 ?		183.0	141.2	.773
346.0	124.0	221.0	120.7	183.0			
R 344.0	173.0	171.0	120.7	141.8	141.7	182.1	.778
344.5	171.0	173.5	122.6	141.6			
D 346.0	119.0	227.0	125.3	181.0	181.2	141.6	.781
346.0	117.5	228.5	126.0	181.3			
R 342.0	174.0	168.0	118.0	142.3	141.5	181.8	.780
342.0	170.5	171.5	122.0	140.8			
D 347.0	130.0	217.0	119.0	182.3	182.5	141.4	.775
347.0	134.0	213.0	116.7	182.7			
R 341.5	174.5	167.0	118.0	141.3	141.2		
341.0	173.0	168.0	119.0	141.1			
					Average		0.776

TABLE X.

Reflection Coefficients in Percentages.								
	C_s.				C_g.			
	Through Air.	Red Glass.	Water.	Diffuse.	Through Air.	Red Glass.	Water.	Diffuse.
	92.8	94.5	88.9	0.98	77.8	75.9	80.8	
	89.8	90.8	86.0	0.92	77.6	76.6	80.0	1.6
	90.8			1.23				
Av'ge	91.1	92.7	87 5	1.04	77.7	76.3	80.4	1.6
	D_s.				D_g.			
	95.0	96.3	91.5	2.2	77.6	76.5	81.0	2.8
	92.0	94.0	90.4		76.7	75.2	79.7	2.2
	94.8	95.0	92.3	0.8				
Av'ge	93.9	95.1	91.4	1.5	77.2	75.9	80.4	2.5
Average Reflection.								
	Air-Silver.				Glass-Silver.			
	92.5	93.9	89.5	1.3	77.5	76.1	80.4	2.0
Corrected Reflection Coefficients.								
	92.0	93.4	89.0		77.6	76.2	80 5	

Average Coefficients through Air, *84.8;* Red Glass, *84.8;* Water, *84.8.*

an angle θ with the incident radiation, then $I_\theta\, \partial A = I_0 \cos\theta\, \partial A$. The total amount of diffuse radiation $= \int\int I_\theta \cos\theta\, \partial A$, over the surface of the hemisphere $= \int_0^{\frac{\pi}{2}} 2\pi r^2\, I_0 \cos\theta \sin\theta\, \partial\theta = \pi\, I_0\, r^2$. This integral is the amount of the diffuse radiation in Table X. The force, due to radiation of intensity $I_\theta\, \partial A$, normal to the vane is $I_\theta \cos\theta\, \partial A$, and the total is equal to $\int_0^{\frac{\pi}{2}} 2\pi r^2\, I_0 \cos^2\theta \sin\theta\, \partial\theta = \frac{2}{3}\, \pi\, I_0\, r^2$. It is thus seen that of the diffuse reflection, two-thirds is effective as light pressure. This increases the air-silver reflection coefficients by 0.9 per cent and the glass-silver values by 1.3 per cent. The small glass rod *d* (Fig. 2), not present in the reflection measurements, decreased the reflecting area of the silvered surfaces in the pressure measurements by 1.54 per cent. The air-silver values are thus decreased by $0.92 \times 1.54 = 1.4$ per cent, and the glass-silver values by $0.78 \times 1.54 = 1.2$ per cent. The application of these two corrections gives the final corrected coefficients in Table X. The diffuse reflection of black coatings deposited by the method used in blackening the silver disc was measured and computed in the same manner as the diffused reflection from the vanes *C* and *D*. The agreement found by Ångström* between the diffuse reflection of matte surfaces for normal incidence and the cosine law was abundantly close for the present purpose. Five determinations of this reflection were made under different conditions and with different coatings. The values in percentages of the incident beam were 4.4 per cent, 4.5 per cent, 4.2 per cent, 4.6 per cent, and 5.2 per cent; average, 4.6 per cent. Thus only 95.4 per cent of the incident beam was absorbed by the black coating on the silver disc in producing the temperature increase observed. Hence the true energy of the beam is equal to the observed energy divided by 0.954.

The silver disc, diameter 13.3 mms., used in the energy measurements, received long waves and scattered radiation which passed round and through the light pressure vanes of diameter 12.8 mms. This amount was experimentally determined for both thin and thick silver coatings in order to approximate to the average condition of the coatings in the light pressure measurements and it was found to average (*a*) through air, 1.40 per cent; (*b*) through red glass, 1.44 per cent; (*c*) through water,

* K. Ångström, Wied. Ann., XXVI. 271 (1885).

0.46 per cent. On this account the energy E of the standard radiation must be reduced by the above percentages. Applying these corrections * and the corrections due to the diffused radiation from the black coating on the silver disc, the energy of the standard radiation becomes

(a) through air, $E_a \times \dfrac{0.986}{0.954}$;

(b) through red glass, $E_g \times \dfrac{0.986}{0.954}$;

(c) through water, $E_w \times \dfrac{0.995}{0.954}$.

Hence the pressure produced by standard radiation calculated by Maxwell's formula, $p = \dfrac{E(1+\rho)}{3 \times 10^{10}}$, since $\rho = 0.848$, becomes

$$\text{(a) through air } = E_a \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954}$$

$$= 1.108 \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954} \times 10^{6} \text{ dynes}$$

$$= (7.05 \pm 0.03) \times 10^{-5} \text{ dynes};$$

$$\text{(b) through red glass } = E_g \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954}$$

$$= 1.078 \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954} \times 10^{6} \text{ dynes}$$

$$= (6.86 \pm 0.03) \times 10^{-5} \text{ dynes};$$

$$\text{(c) through water } = E_w \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.995}{0.954}$$

$$= 1.008 \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.995}{0.954} \times 10^{6} \text{ dynes}$$

$$= (6.48 \pm 0.04) \times 10^{-5} \text{ dynes}.$$

* As the average pitch of the cone of the incident beam was about one part in forty, no correction need be applied for inclination. Furthermore, the inside of the bell-jar was blackened and the zero of the balance was so chosen that energy reflected from the window admitting the beam could produce no pressure effects.

A comparison of observed and computed pressures follows: —

	Observed values in 10^{-5} dynes.	Computed values in 10^{-5} dynes.	Obs.-comp. in percentages.
Through air,	$p = 7.01 \pm 0.02$ *	7.05 ± 0.03	-0.6
Through red glass,	$p = 6.94 \pm 0.02$	6.86 ± 0.03	$+1.1$
Through water,	$p = 6.52 \pm 0.03$	6.48 ± 0.04	-0.6

An estimate of the approximate magnitude of the gas action, not eliminated by the ballistic method of observation, may be reached from the following considerations.

When radiation falls upon a vane of the torsion balance, part of it is absorbed by the silver surface. From the amounts directly and diffusely reflected, as given in Table X, the amount transmitted by the average surface (experimentally determined but not given in Table X), the effect of the glass rod and the reflection coefficient of the glass surface, it was found that, when the silver side of the vane was toward the radiation source, the absorption coefficient for radiation through air was 6 per cent, and when the glass surface was forward, it was 18 per cent.

The total force acting on the vane is made up of two parts, that due to radiation pressure and that due to gas action. Let F_r be the force due to the first cause, assuming that all the radiation is absorbed, and F_g the effect due to the second, on the same condition. Then the total effect, when the silver side of the vane is forward and the radiation is "through air," is $1.92\ F_r + 0.06\ F_g$. When the glass side is forward the total effect is $1.776\ F_r - 0.18\ F_g$. Making these expressions equal to the reduced deflections (Table III, columns 11 and 12) on the silver and glass surfaces respectively, we have two equations by means of which the values of F_r and F_g may be obtained. Hence the effect due to gas action on each face of the vane is approximately determinate, as is

* The pressure and energy measurements for the three different wave groups through air, red glass, and water cell, constitute three independent experiments. In the values for pressure, 7.01, 6.94, and 6.52, equality is not to be looked for. The difference arises from the different reflecting power of the 45° glass plate (Fig. 3) for the different beams and from the fact that the indications of the lamp galvanometer G_2 connected with the bolometer R, were probably not strictly proportional to energy for throws differing as widely as 33, 60, and 100, which, roughly, were the relative intensities of the beams through water cell, red glass, and air. The function of the lamp bolometer and galvanometer was purely to keep a check on the small variations of the lamp which rarely fluctuated more than 10 per cent on either side of the mean value.

also the part (0.06 F_g) not eliminated when we average the two columns to obtain column 13.

Applying this method to all the results of Table III (with the exception of those results taken with poor mirrors as shown by our notes), the gas action present in the ballistic deflections "through air" is 0.8 per cent. Applying the corresponding data and equations to Table IV, the gas action present in the red glass values is 1.1 per cent and in the water cell values, 0.3 per cent. The sign of F_g comes out negative, which means that the gas action was suction.

This reasoning assumes that the glass faces of the vanes during the six seconds exposure are not warmed by absorption nor by the conduction of heat through the thin glass from the silver coating. The effect of any such absorption or conduction would be to diminish the computed gas action. As estimated from the static observations, the gas action in the ballistic measurements is comparable in magnitude with the computed values obtained above, and of the same sign. Both results show that the uneliminated gas action by the most liberal estimate cannot have exceeded 1 per cent of the radiation pressure. Because of its smallness and indefiniteness no correction for gas action has been made to the final pressure values. If corrections were applied its effect would be to slightly reduce the observed pressures.

Aside from the measurements of pressure and energy for which the probable errors are given, the percentage accuracies in the other measurements entering into the computations, and their effects upon the final result follow: —

1. Quantities which affect individual series:

(*a*) Pressure values, —

Period of balance	T, accurate to	0.2% ;	effect on	result	0.0%
Lever arm of balance	l, "	0.1% ;	"	"	0.0%
Constant of galv'meter	G_2, "	0.5% ;	"	"	0.0%
Estimate of possible error due to changing ratio of period of G_2 to length of exposure of bolometer		0.4% ;	"	"	0.1%

(*b*) Energy values, —

Constant of galv'meter	G_1, "	0.1% ;	"	"	0.0%
" "	G_2, "	0.5% ;	"	"	0.0%

2. Quantities which affect final averages:

(*a*) Pressure values, —

Torsion of fibre,	accurate to	0.2% ;	effect on result		0.2%
Reducing factor, 1.357	"	0.1% ;	"	"	0.1%
Reducing factor, 1.550 for G_2	"	0.2% ;	"	"	0.2%
Reflection of surfaces of vanes	"	0.4% ;	"	"	0.2%

(*b*) Energy values, —

Mass of silver disc	"	0.1% ;	"	"	0.1%
Thermal calibration of disc	"	0.5% ;	"	"	0.5%
Diffuse reflection black coating	"	5.0% ;	"	"	0.1%

From the agreement within the probable error of the air, red glass, and water values with the theory, it appears that the radiation pressure depends only upon the intensity of the radiation and is independent of the wave length.

The Maxwell-Bartoli theory is thus quantitatively confirmed within the probable errors of observation.

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